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Open Innovation in a Dynamic Cournot Duopoly

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Abstract

In recent years Open Innovation (OI) processes have been receiving growing attention from the empirical and theoretical economic literature, where a debate is taking place on the aspects of complementarity or substitutability between internal R&D and OI spillover. By means of a differential game approach, we analyze the case of substitutability in an OI setup in a Cournot duopoly where knowledge spillovers are endogenously determined via the R&D process. The game produces multiple steady states, allowing for an asymmetric solution where a firm may trade off the R&D investment against information absorption from the rival. The technical analysis and the numerical simulations point out that the firm which commits to a higher level of OI absorption produces a smaller output and enjoys higher profits than its rival.

JEL codes: C73, L13, O31

Keywords: R&D, spillovers, dynamic games

1 Introduction

According to Chesbrough (2003), the economic system is entering a new era of Open Innovation (OI), where OI is defined as "the use of purposive inflows and outflows of knowledge to accelerate internal innovation, and expand the markets for external use of innovation, respectively". This fact is a logical consequence of a fast growing and highly competitive market for new technologies. At this rate of growth, the companies' internal resources are not sufficient to meet the new challenges, and they have to access external sources. What drives firms towards OI is the fact that many companies are obliged to innovate and develop new products under extremely tough time and resource constraints in order to stay in the market and keep being competitive.

OI is vital for companies whose products have short life cycles (for example software and consumer electronics) and extremely demanding criteria regarding quality, price and customers' expectations. Specifically, the typical example of OI is the Open Source software: Linux, Unix, Mozilla, OpenOffice, Android are products whose source code has been partially or completely released (for an exhaustive overview of the economics of Open Source and on its technological aspects, see Lerner and Tirole, 2002, and West and Gallagher, 2006). Open Source developed by Information Technology firms may be a convenient strategy, when companies expect to boost their profits in a complementary segment, or when they are so small that they cannot compete commercially in the primary segment (Lerner and Tirole, 2002).

The complexity and diversity of the knowledge structure, like nanotechnologies and biotechnologies, are another important factor. As long as "not all smart people work for you" (Chesbrough 2003), there is an increasingly dispersed distribution of useful knowledge in companies of all sizes. The amount of available information is heterogeneously spread and companies cannot access and monitor all necessary networks. Those networks are built on several factors, including governmental and private organizations, academic research, synthetic knowledge, know-how and highly specialized knowledge based on experience and interaction among the agents. Hence, OI provides an access to these networks. Thus, in a fast developing and expanding environment, relying on the Closed Innovation approach is both insufficient and risky.

The OI policy allows the company to use external technologies (Spithoven *et al.*, 2011) and share its own knowledge with outside partners at a strictly managed and controlled level. The company boundaries become "permeable" (Mortara *et al.*, 2009). The process of learning, accumulation of outside knowledge and competence enables the company to be not only more innovative, but also innovate at a higher speed. It creates linkages among companies that stimulate the sharing of ideas, technology and experience. These strategies, as presented by Monica Beltrametti, Vice President of the Xerox Research Center Europe (XRCE) at Grenoble Innovation Fair (GIF) on October 2009, help share the risk, and reduce costs by using more suppliers. This also opens the way to new markets and explorations (ideas from new external participants, such as governmental organizations, universities, other large companies and individual representatives).

Furthermore, Open Innovation is an innovation in itself (Mortara *et al.*, 2009). It stimulates innovation by distributing the cost and risk, offering access to new contact networks through "intermediaries" (Mortara, 2010) and information pools, and also internalizes the unintended byproducts of innovation process, spillovers (Bogers, 2011). A recent study on the underlying factors affecting the degree of openness of a firm from a strategic point of view is provided by Drechsler and Natter (2012). An accurate description of the various kinds of OI and on the way it has been treated in literature can be found in Dahlander and Gann (2010). Assuming the distinction between inbound and outbound OI, they identify sourcing and acquiring as the typical inbound processes and revealing and selling as the typical outbound processes and investigate the advantages and the disadvantages of each form of openness.

The Closed Innovation model based on the traditional patent system discards unintended or irrelevant research results; however, it has long been considered as a necessary cost for enhancing technical progress. Conversely, the OI policy allows the company to sell those products that do not meet the firm's capacity and development possibilities (in form of IP licenses) to other companies (Chesbrough 2003, Chesbrough *et al.* 2006). For example, in bio-pharmaceutical industry firms out-license for profit in different stages of R&D process, because they endogenously decide how much of innovation they want to disclose or sell (Bianchi *et al.*, 2011). If observed from this new angle, the firm turns out to have two functions: one is learning from the environment and the other is constructing it by sharing its own knowledge via a systematic spillover flow. Hence, no proof exists that OI generates less spillovers. The company must spend in order to innovate, it must create and achieve itself the new technologies or find them on the market in the opposite case. Thus, there should be a balance between the level of internal R&D and external knowledge resources accrued by the firms. Otherwise, due to spillover's negative effect, that reduces the gains, the firms have a lower investment level than socially desired (as in Arrow, 1962, *inter alia*). In other cases firms may not take into account the positive level their spillovers have on other firms' R&D and *vice versa*, leading again to suboptimal level of R&D (Romer, 1990). On the other hand, firms may be trapped into patent racing and overinvest, thus significantly affecting their profits levels. Moreover, they can disclose too much internal information by adopting a purely Open Innovation approach (Enkel *et al.*, 2009).

An important issue about OI is the ongoing debate on the complementarity or substitutability between the internal and external knowledge used for innovation in a given company. In order to maximize the profits and achieve better results, the company should be able to efficiently scan the environment and correctly assess the degree of complementarity between its R&D program and externally available technology in order to take advantage from the OI policy (Vanhaverbeke *et al.*, 2008). This aspect of a firm's overall strategy can be thought of as a dynamic capability which is built over time (Helfat *et al.*, 2007). Despite these arguments, there are scholars proving that, strategically, some firms rely on substitutability rather than complementarity, due to higher costs of the latter, industry specific qualities or budget constraints.

Moreover, it is worth stressing that OI can be achieved through different routes. One possibility is that the firm implements the policy as a "conscious" movement due to its internal necessities (Mortara *et al.*, 2009). Another possibility is that firms are pushed towards OI by some external factors, like globalization, knowledge-intensive environment, markets, or customer preferences. What happens if the firms are able to optimally determine the spillover delivered to others in the industry, and - in turn - rationally grab the spillovers created by other firms? The effect is an increase in their gains, and a decrease in unnecessary competitiveness. According to Jaffe (1986), who analyzed the relevance of external R&D to individual companies, spillovers have a positive impact on productivity of own R&D, and a negative effect on competitiveness. This effect is confirmed in a recent empirical paper by Czarnitzki *et al.* (2012).

Since OI has been studied theoretically and empirically in the latest two decades, some models have already appeared in the economic literature, especially concerning Open Source software. The most common frameworks have been the static oligopoly games by far. Some aspects of technology transfers and differentiated Cournot duopolies leading to asymmetric equilibrium structures have been investigated by Zanchettin (2006), whereas Li and Ji (2010) have focused on the characteristics of cost-reducing innovation. Modica (2012) has proposed a model of Open Source development as a two-stage oligopoly game. Another two-stage game is the one constructed by Llanes and de Elejalde (2013) between Open Source and proprietary firms. To the best of our knowledge, one of the very few contributions on OI building on a dynamic model has been conceived by Caulkins *et al.* (2013), where the authors characterize the optimal strategy of a firm when it has to choose between proprietary and Open Source software. From a technical viewpoint, their model is an optimal control problem for a firm in the market subject to the evolutionary dynamics of the quality of its product. Their analysis focuses on the crucial role of R&D costs, and intends to explain when it is convenient to open the source code for the firm.

But why does a large company choose an OI policy? A very relevant example of large IT company which committed to OI development strategies is IBM. In 1993 IBM agreed to sell its industry-leading $2\frac{1}{2}$-inch drives to Apple, which was one of its direct competitors, to use inside its PowerBook laptop computers (see Chesbrough, 2003). In 1999, IBM announced support for Linux, starting to invest financial and technical resources to foster growth, development and use of Linux Open Source technology (see the IBM website for official information). In particular, IBM simply explains its commitment to Open Source technology by listing some clear motivations: 'IBM established the Linux Technology Center (LTC) as the primary vehicle to participate in the Linux community. IBM and the LTC have established four goals for participation in the Linux community: make Linux better, expand Linux's reach for new workloads, enable IBM products to operate with Linux, increase collaboration with customers to innovate in ways IBM cannot do by itself'.

In this paper, we will take into account a duopoly allowing for both symmetric and asymmetric optimal strategies where the two firms engage in a Cournot competition to maximize their discounted payoff flows over an infinite horizon. In our setup, the firms' strategic variables are the quantities and the levels of R&D efforts, whereas the state variables are the marginal costs of production and the levels of positive technological spillover spreading from a firm to its rival. Namely, we will look upon OI for one player as depending on her opponent's development of R&D, as if OI was the fraction of the research she takes or buys from outside, then resulting in a sort of substitute (not complement) with respect to her own R&D. Consequently, we will focus on the aspects of substitutability of OI rather than complementarity of OI. In this respect, Spithoven *et al.* (p. 240, 2012) state that 'However, empirical evidence does not always support the idea of complementarity between internal and external R&D. Based on a cross-section of Dutch manufacturing firms, Audretsch *et al.* (1996) reported a substitution effect of external R&D activity in low and medium technology industries (the reverse was true in high-tech industries)'. Also Fu (2012) finds that the complementarity between internal R&D and OI is not always feasible, being too costly. It is relevant to remark that OI is not totally free, i.e. we will take into account an appropriation cost of external innovation for the firms.[1]

The model we adopt builds upon Cellini and Lambertini (2005, 2009), with respect to which there are two fundamental differences. First, the knowledge spillovers are endogenously determined during the R&D process, because they are state variables which evolve over time as they are affected by the R&D efforts. This is a clear-cut complication of Cellini and Lambertini (2005, 2009), where the technology spillover is described by a constant parameter, identical across firms. Secondly, the model is extended in that we are not going to impose any symmetry assumption, thus allowing for the presence of both symmetric and asymmetric optimal trajectories and steady states. This is actually helpful to separately assess the results between the firm which spends more or less in internal R&D or in OI absorption.

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[1] This aspect is confirmed by some empirical papers such as Lokshin *et al.*(2008) and Fu (2012), which have analyzed the impact of internal and external R&D on productivity and incentives for further innovation. In particular, Lokshin *et al.* (2008) have examined the data on a 6-year panel of the Dutch manufacturing firms, finding that the costs of appropriation of external R&D were definitely smaller than the costs of development of internal R&D.

Besides being an improvement of a previous differential game, this model intends to shed some light on the effects of firms' strategic behavior when they can acquire both internal and external R&D resources. Given the substitutability of R&D and OI, it is interesting to assess the output levels and the profit levels at equilibrium so as to identify the most convenient investment strategy in a small market. The main findings of the paper can be outlined as follows:

- We prove the existence of multiple steady states, in particular a symmetric and an asymmetric one. In the asymmetric steady state, a firm may trade off the R&D investment against information absorption from the rival.
- We deduce from the technical results and from the numerical simulation that, interestingly, the firm investing less in R&D enjoys higher profits than the rival, thanks to the combining effects of savings upon investment costs and exploiting information transmission.
- The firm with a higher private R&D investment level can commit to a definitely smaller level of OI appropriation effort, and *vice versa*.

The structure of the paper includes Section 2, where the setup of our model is outlined, Section 3, containing the analytical study of the optimal strategies, and separately featuring the symmetric and the asymmetric case. Section 4 includes our conclusions and supplies hints for future developments.

2 The Setup

We consider an infinite horizon differential game modeling a duopoly with single-product firms in continuous time $t \in [0, \infty)$. For simplicity, assume firms supply homogeneous goods. Define the inverse market demand function as

$$p(t) = A - q_1(t) - q_2(t), \tag{1}$$

where $A > 0$ is the constant reservation price and $q_i(t)$ is the quantity produced by the i-th firm at time t.

Production takes place at constant returns to scale, with marginal cost $c_i(t)$ evolving over time according to the following dynamic equation:

$$\frac{dc_i}{dt} \equiv \dot{c}_i(t) = -k_i(t) - \beta_i(t)k_j(t) + \delta c_i(t),\ i, j = 1, 2; i \neq j \tag{2}$$

where $k_i(t)$ is the R&D effort exerted by firm i at time t, and $\beta_i(t) \in (0, 1)$ is the level of positive technological spillover or the level of OI enjoyed by firm i (and transmitted by firm j).[2] Parameter $\delta \geq 0$ is a constant depreciation rate that results in decreasing returns due to aging of the technology.

Our approach to spillovers is precisely the feature of the model where we depart from Cellini and Lambertini (2005, 2009). Here, we allow for the spillover to be endogenously determined by the firms' instantaneous R&D efforts, whereby Open Innovation made available to firm i changes over time according to the following dynamic equation:

$$\frac{d\beta_i}{dt} \equiv \dot{\beta}_i(t) = \alpha k_j(t) - \eta \beta_i(t) \tag{3}$$

where α and η are positive parameters. The above equation refers to a situation where firm i has access to an amount of OI (a state variable) that deteriorates if firm j ceases to carry out any R&D.

(3) deserves some further explanations: this representation implies that the inflow of knowledge from the i-th firms stems from the R&D carried out by the j-th firm and *vice versa*. By acquiring, or buying, such knowledge from its opponent, a firm may substitute its own R&D with OI and the inflow of knowledge for one agent corresponds to the outflow for the other agent. Hence, this kinematic equation describes our idea of substitutability between R&D and OI, also corresponding to the endogenization of the spillover effect, contrary to Cellini and Lambertini (2005, 2009), where it is a constant parameter. α, which will be taken as the relevant parameter for the analysis of the equilibrium profits in Subsections 3.1.1 and 3.2.1, represents the rate at which the spillover effect on OI grows as it is driven by external R&D.

The cost of creating R&D by firm i is described by the convex function: $\Gamma_i(k_i(t)) = \dfrac{b[k_i(t)]^2}{2}$, where b is a positive parameter. We are also assuming that, in order for a firm to be able to absorb positive externalities from the environment, it has to bear some appropriation cost, which can be represented as $C_i(\beta_i(t)) = \dfrac{\epsilon[\beta_i(t)]^2}{2}$, where ϵ is a positive parameter. For example, it can be generated by the process of searching and assimilating new knowledge, and subsequently adapting it to the firms necessities and standards.

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[2] In this model, productive technologies are perfect substitutes. For an alternative approach where complementarity is considered, see Scotchmer (2010).

Accordingly, the instantaneous profit function of the i-th firm will be written as follows:

$$\pi_i(t) = (p(t) - c_i(t))q_i(t) - \Gamma_i(k_i(t)) - C_i(\beta_i(t)) =$$

$$= [A - q_i(t) - q_j(t) - c_i(t)]q_i(t) - \frac{b[k_i(t)]^2}{2} - \frac{\epsilon[\beta_i(t)]^2}{2}.$$

We shall analyze a fully noncooperative game in which each firm sets independently its own level of R&D (determining thus their productive efficiency and the respective degrees of information sharing), as well as the level of output it wants to sell. Hence, the problem of firm i is to

$$\underset{q_i,k_i \geq 0}{max}\ \Pi_i \equiv \int_0^\infty \pi_i(t)e^{-\rho t}\,dt \tag{4}$$

subject to

$$\dot{c}_i(t) = -k_i(t) - \beta_i(t)k_j(t) + \delta c_i(t) \tag{5}$$

$$\dot{c}_j(t) = -k_j(t) - \beta_j(t)k_i(t) + \delta c_j(t) \tag{6}$$

$$\dot{\beta}_i(t) = \alpha k_j(t) - \eta\beta_i(t) \tag{7}$$

$$\dot{\beta}_j(t) = \alpha k_i(t) - \eta\beta_j(t) \tag{8}$$

whose initial conditions are $c_i(0) = c_{i0} \geq 0$, $\beta_i(0) = \beta_{i0} \geq 0$, for $i = 1, 2$. Along the game firms will discount the future profits, hence $\rho > 0$ is an intertemporal discount rate common to both firms.

3 The game

The Hamiltonian of the i-th player is a function depending on the arguments: t, q_1, q_2, k_1, k_2, c_1, c_2, β_1, β_2 and on all the related costate variables. In turn, all arguments are dependent on time, but from now on we will omit the time arguments whenever possible to simplify notation. The current value Hamiltonian function $\mathcal{H}_i$ takes the following form:

$$\mathcal{H}_i(\cdot) = e^{-\rho t}\Bigg\{[A - q_i(t) - q_j(t) - c_i(t)]q_i(t) - \frac{b[k_i(t)]^2}{2} - \frac{\epsilon[\beta_i(t)]^2}{2} +$$

$$+\lambda_{ii}(t)[-k_i(t) - \beta_i(t)k_j(t) + \delta c_i(t)] + \lambda_{ij}(t)[-k_j(t) - \beta_j(t)k_i(t) + \delta c_j(t)] +$$

$$+\mu_{ii}[\alpha k_j(t) - \eta\beta_i(t)] + \mu_{ij}[\alpha k_i(t) - \eta\beta_j(t)]\} \tag{9}$$

where $\lambda_{ii}(t)$ and $\lambda_{ij}(t)$ are the current value co-state variables respectively associated to the state variables $c_i(t)$ and $c_j(t)$, and $\mu_{ii}(t)$ and $\mu_{ij}(t)$ are the current value co-state variables respectively associated to the state variables $\beta_i(t)$ and $\beta_j(t)$.

We are going to determine the open-loop information structure of this game by applying the Pontryagin's Maximum Principle. The procedure we are going to implement is standard in differential game theory applied to industrial economic models (analogous techniques can be found in Cellini and Lambertini, 1998, 2002, 2009).

The first-order conditions (FOCs) are

$$\frac{\partial \mathcal{H}_i}{\partial q_i} = 0 \implies A - 2q_i - q_j - c_i = 0 \tag{10}$$

$$\frac{\partial \mathcal{H}_i}{\partial k_i} = 0 \implies -bk_i - \lambda_{ii} - \beta_j \lambda_{ij} + \alpha \mu_{ij} = 0 \tag{11}$$

Note that the linear-quadratic structure of the Hamiltonian function ensures the concavity w.r.t. the control variables, and then the existence of a maximum point; indeed, the second order conditions read as:

$$\frac{\partial^2 \mathcal{H}_i}{\partial q_i^2} = -2 < 0 \tag{12}$$

$$\frac{\partial^2 \mathcal{H}_i}{\partial k_i^2} = -b < 0 \tag{13}$$

Before carrying out the standard procedure, we must point out that this game has degenerate features, in that (10) do not contain any costate variable. Hence, (10) represent a couple of policy functions which will be employed to subsequently determine the equilibrium structure. Differentiating (10) w.r.t. time we obtain:

$$\dot{c}_i(t) = -2\dot{q}_i(t) - \dot{q}_j$$
$$\dot{c}_j(t) = -2\dot{q}_j(t) - \dot{q}_i$$

Plugging the expressions obtained from the latter relations and from (10) into (5) and (6), we achieve the dynamic equations:

$$-2\dot{q}_i(t) - \dot{q}_j = -k_i(t) - \beta_i(t)k_j(t) + \delta c_i(t) \tag{14}$$

$$-2\dot{q}_j(t) - \dot{q}_i = -k_j(t) - \beta_j(t)k_i(t) + \delta c_j(t) \tag{15}$$

From (14) and (15) we deduce the following output dynamics:

$$\dot{q}_i(t) = \frac{1}{3}[(2-\beta_j(t))k_i(t) + (-1+2\beta_i(t))k_j(t) + \delta(c_j(t) - 2c_i(t))] \tag{16}$$

$$\dot{q}_j(t) = \frac{1}{3}[(-1+2\beta_j(t))k_i(t) + (2-\beta_i(t))k_j(t) + \delta(-2c_j(t) + c_i(t))] \tag{17}$$

By construction, (16) and (17) depend linearly on (5) and (6), so they are not going to provide any further information about equilibrium. The adjoint equations and transversality conditions for the costates λ_{ij} amount to:

$$\begin{cases} \dot{\lambda}_{ii}(t) = (\rho-\delta)\lambda_{ii}(t) + q_i(t) \\ \\ \lim_{t\to+\infty} e^{-\rho t}\lambda_{ii}(t)c_i(t) = 0 \end{cases}, \tag{18}$$

$$\begin{cases} \dot{\lambda}_{ij}(t) = (\rho-\delta)\lambda_{ij}(t) \\ \\ \lim_{t\to+\infty} e^{-\rho t}\lambda_{ij}(t)c_j(t) = 0 \end{cases}. \tag{19}$$

for $i \neq j$, (19) admits the solutions $\lambda_{ij} \equiv 0$. The adjoint equations and transversality conditions for the costates μ_{ij} are:

$$\begin{cases} \dot{\mu}_{ii}(t) = \epsilon\beta_i(t) + \lambda_{ii}(t)k_j(t) + (\rho+\eta)\mu_{ii}(t) \\ \lim_{t\to+\infty} e^{-\rho t}\mu_{ii}(t)\beta_i(t) = 0 \end{cases}, \tag{20}$$

$$\begin{cases} \dot{\mu}_{ij}(t) = \lambda_{ij}(t)k_j(t) + (\rho+\eta)\mu_{ij} \\ \\ \lim_{t\to+\infty} e^{-\rho t}\mu_{ij}(t)\beta_j(t) = 0 \end{cases}. \tag{21}$$

Plugging the solutions to (19) into (21) implies that $\mu_{ij}(t) \equiv 0$ are solutions to (21), for $i \neq j$.

Before proceeding any further, some technical aspects are to be explained in detail. The peculiarity of this game relies upon the fact that, since the costate variables μ_{ii}, for $i = 1, 2$, do not enter the FOCs, they are not actually relevant for equilibrium. The standard procedure, i.e. the differentiation w.r.t. time of (10) and of (11) can only yield a system of control equations originated from (18). The corresponding policy implication suggests that the diffusion of the spillover does not affect the optimal policy of the i-th firm, so that the spillover effects are completely exogenous with respect to the producer's strategy. In other words, although the beneficial consequences of OI on profit flows are obviously affecting both firms, the dynamics of its evolution is irrelevant on the strategic decisions.

Therefore, we will only take into account the transversality conditions for λ_{ii}. Since the involved optimal states and costates are:

$$c_i^*(t) = \left(c_i^*(0) - \int_0^t (k_i^*(s) + \beta_i^*(s) k_j^*(s)) e^{-\delta s} ds \right) e^{\delta t},$$

$$\lambda_{ii}^*(t) = \left(\lambda_{ii}^*(0) + \int_0^t q_i^*(s) e^{(\delta - \rho)s} ds \right) e^{(\rho - \delta)t},$$

then the transversality conditions hold if and only if:

$$\lim_{t \to +\infty} \left(\lambda_{ii}^*(0) + \int_0^t q_i^*(s) e^{(\delta-\rho)s} ds \right) \left(c_i^*(0) + \int_0^t (k_i^*(s) - \beta_i^*(s) k_j^*(s)) e^{-\delta s} ds \right) = 0,$$

i.e.

$$\lambda_{ii}^*(0) = - \int_0^\infty q_i^*(s) e^{(\delta-\rho)s} ds,$$

then the optimal costates must be:

$$\lambda_{ii}^*(t) = - \left(\int_t^\infty q_i^*(s) e^{(\delta-\rho)s} ds \right) e^{(\rho-\delta)t}.$$

The output control equations will follow from the differentiation of (10), entailing that they are linearly dependent on the dynamic constraints. Subsequently, we achieve the simple identity:

$$\dot{\lambda}_{ii}(t) = -b \dot{k}_i(t), \tag{22}$$

then, exploiting (11) and (22) in (18) we have:

$$\dot{k}_i(t) = (\rho - \delta) k_i(t) - \frac{q_i(t)}{b} \tag{23}$$

$$\dot{k}_j(t) = (\rho - \delta) k_j(t) - \frac{q_j(t)}{b} \tag{24}$$

leading to the following state-control dynamical system, consisting in eight ordinary differential equations:

$$\begin{cases} \dot{c}_i(t) = -k_i(t) - \beta_i(t)k_j(t) + \delta c_i(t) \\ \\ \dot{c}_j(t) = -k_j(t) - \beta_j(t)k_i(t) + \delta c_j(t) \\ \\ \dot{\beta}_i(t) = \alpha k_j(t) - \eta\beta_i(t) \\ \\ \dot{\beta}_j(t) = \alpha k_i(t) - \eta\beta_j(t) \\ \\ \dot{k}_i(t) = (\rho-\delta)k_i(t) - \frac{q_i(t)}{b} \\ \\ \dot{k}_j(t) = (\rho-\delta)k_j(t) - \frac{q_j(t)}{b} \\ \\ \dot{q}_i(t) = \dfrac{1}{3}[(2-\beta_j(t))k_i(t) + (-1+2\beta_i(t))k_j(t) + \delta(c_j(t) - 2c_i(t))] \\ \\ \dot{q}_j(t) = \dfrac{1}{3}[(-1+2\beta_j(t))k_i(t) + (2-\beta_i(t))k_j(t) + \delta(-2c_j(t) + c_i(t))]. \end{cases} \tag{25}$$

We are now going to search for the possible steady states of (25) by letting all its equations vanish, but the aforementioned linear dependence of $\dot{q}_i(t)$ and of $\dot{q}_j(t)$ leaves us with only six equations and eight unknowns. Consequently, we will make use of (10), thus expressing all state variables depending on the R&D efforts as follows:

$$c_i = A - b(\rho-\delta)(k_j + 2k_i), \tag{26}$$

$$c_j = A - b(\rho-\delta)(k_i + 2k_j), \tag{27}$$

$$\beta_i = \frac{\alpha}{\eta}k_j, \tag{28}$$

$$\beta_j = \frac{\alpha}{\eta}k_i. \tag{29}$$

By replacing the resulting expressions for c_i, c_j, β_i and β_j in the last two equations of (25) and imposing stationarity, we obtain the following two nonlinear equations in k_i and k_j:

$$-k_i - \frac{\alpha}{\eta} k_j^2 + \delta\left[A - b(\rho-\delta)(k_j + 2k_i)\right] = 0 \tag{30}$$

$$-k_j - \frac{\alpha}{\eta} k_i^2 + \delta\left[A - b(\rho-\delta)(k_i + 2k_j)\right] = 0 \tag{31}$$

Then, subtracting (31) from (30) and subsequently factoring the equation:

$$k_j - k_i - \frac{\alpha}{\eta}\left(k_j^2 - k_i^2\right) + \delta b(\rho-\delta)\left[k_j - k_i\right] = 0 \iff$$

$$\iff (k_j - k_i)\left[1 - \frac{\alpha}{\eta}(k_j + k_i) + \delta b(\rho-\delta)\right] = 0,$$

we obtain two kinds of different zeros:

$$k_i = k_j \qquad \text{and} \qquad k_i = \frac{(1 + b\delta(\rho-\delta))\eta}{\alpha} - k_j. \tag{32}$$

In the following Subsections, we shall investigate both the symmetric and the asymmetric equilibrium cases.

3.1 The symmetric equilibrium structure

The following Proposition illustrates the properties of the symmetric equilibrium point:

Proposition 1. *If*

1. $\rho > \delta$;
2. $\alpha < \frac{\eta}{\delta A}[2 + 3b\delta(\rho-\delta)]$,

the game admits a symmetric steady state $P = (c_1^*, c_2^*, \beta_1^*, \beta_2^*, k_1^*, k_2^*, q_1^*, q_2^*)$,[3] *where:*

$$k_1^* = k_2^* = k^* = \frac{-\eta[1 + 3b\delta(\rho-\delta)] + \sqrt{\eta^2(1 + 3b\delta(\rho-\delta))^2 + 4\alpha\eta\delta A}}{2\alpha},$$

---

[3] Note that under symmetry, the 2 dynamics (3) collapse into a unique one, and the respective R&D efforts cannot be distinguished, then it may also be interpreted as the case in which the i-th firm's spillover is produced by its own R&D and not from its competitor's R&D.

$$c_1^* = c_2^* = c^* = A - 3b(\rho - \delta)k^*,$$
$$\beta_1^* = \beta_2^* = \beta^* = \frac{\alpha}{\eta}k^*,$$
$$q_1^* = q_2^* = q^* = b(\rho - \delta)k^*.$$

Proof. Consider the symmetric case, i.e. $k_i = k_j = k$ and substitute in (30):

$$-k - \frac{\alpha}{\eta}k^2 + \delta(A - b(\rho - \delta)(k + 2k)) = 0,$$

whose zeros are:

$$k_{1,2}^* = \frac{-\eta[1 + 3b\delta(\rho - \delta)] \pm \sqrt{\eta^2(1 + 3b\delta(\rho - \delta))^2 + 4\alpha\eta\delta A}}{2\alpha}.$$

If $\rho > \delta$, it can be easily observed that the smallest solution is negative, whereas the remaining one is positive, and consequently feasible.

Since the level of spillover β^* is supposed to belong to the interval $(0, 1)$, we need to find suitable assumptions so that this property be satisfied:

$$\beta^* = \frac{\alpha}{\eta}k^* = \frac{-\eta[1 + 3b\delta(\rho - \delta)] + \sqrt{\eta^2(1 + 3b\delta(\rho - \delta))^2 + 4\alpha\eta\delta A}}{2\eta} < 1 \iff$$
$$\iff (\eta(3 + 3b\delta(\rho - \delta)))^2 - \eta(4A\alpha\delta + \eta(1 + 3b\delta(\rho - \delta))^2) > 0 \iff$$
$$\iff \dots \iff \alpha < \frac{\eta}{\delta A}[2 + 3b\delta(\rho - \delta)],$$

hence if $\alpha < \frac{\eta}{\delta A}[2 + 3b\delta(\rho - \delta)]$, then $\beta^* \in (0, 1)$.

Moreover, since the remaining coordinates of P, achieved by the relations originated by the vanishing of (25), are positive for $\rho > \delta$, then P is a feasible steady state for the system (25). □

As far as the dynamic features of the trajectories are concerned, we can get some information from the eigenvalues of the 4 × 4 Jacobian matrix of the dynamic system achieved from the symmetry assumptions, that is:

$$\mathcal{J}(P) = \begin{pmatrix} \frac{\partial \dot{c}}{\partial c} & \frac{\partial \dot{c}}{\partial \beta} & \frac{\partial \dot{c}}{\partial k} & \frac{\partial \dot{c}}{\partial q} \\ \frac{\partial \dot{\beta}}{\partial c} & \frac{\partial \dot{\beta}}{\partial \beta} & \frac{\partial \dot{\beta}}{\partial k} & \frac{\partial \dot{\beta}}{\partial q} \\ \frac{\partial \dot{k}}{\partial c} & \frac{\partial \dot{k}}{\partial \beta} & \frac{\partial \dot{k}}{\partial k} & \frac{\partial \dot{k}}{\partial q} \\ \frac{\partial \dot{q}}{\partial c} & \frac{\partial \dot{q}}{\partial \beta} & \frac{\partial \dot{q}}{\partial k} & \frac{\partial \dot{q}}{\partial q} \end{pmatrix} = \begin{pmatrix} \delta & -k^* & -1 - \beta^* & 0 \\ 0 & -\eta & \alpha & 0 \\ 0 & 0 & \rho - \delta & -\frac{1}{b} \\ -\frac{\delta}{3} & \frac{k^*}{3} & \frac{1+\beta^*}{3} & 0 \end{pmatrix}. \quad (33)$$

Proposition 2. *The symmetric steady state P is a saddle point equilibrium.*

Proof. In order to check the stability of P, we have to evaluate the eigenvalues (or zeros) of the characteristic polynomial of (33). Calling λ the unknown of the equation, we have:

$$p(\mathcal{J}(P)) = (\delta - \lambda)\left[(\eta\lambda + \lambda^2)(\rho - \delta - \lambda) - \frac{\alpha k^*}{3b} - \frac{(1+\beta^*)(\eta + \lambda)}{3b}\right] +$$

$$+\frac{\delta}{3b}[\alpha k^* + (1+\beta^*)(\eta+\lambda)] = \cdots =$$

$$= \lambda(\delta - \lambda)(\eta + \lambda)(\rho - \delta - \lambda) + \lambda\frac{k^* + (1+\beta^*)(\eta+\lambda)}{3b},$$

having the zero $\lambda_1 = 0$. The remaining 3rd degree polynomial admits at least one real negative zero because the known term $\rho\eta(\rho - \delta) + \frac{k^*+(1+\beta^*)\eta}{3b}$ is positive when P exists and is feasible by Proposition 1. Hence, P is a saddle point equilibrium whose stable manifold has at least dimension 1. □

This analysis ensures that there exist trajectories heading towards that equilibrium point and also trajectories running away from it.

The expression of profit evaluated at P, expressed by means of k^*, is[4]

$$\Pi^* = (A - 2q^* - c^*)q^* - \frac{b(k^*)^2}{2} - \frac{\epsilon(\beta^*)^2}{2} =$$

$$= \cdots = (k^*)^2\left[b^2(\rho-\delta)^2 - \frac{b}{2} - \frac{\epsilon\alpha^2}{2\eta^2}\right]. \tag{34}$$

The next Proposition intends to characterize the assumptions for the positivity of (34).

Proposition 3. *If*

1. $\rho > \delta$;
2. $b > \dfrac{1}{2(\rho-\delta)^2}$;
3. $\alpha < min\left\{\dfrac{\eta}{\delta A}[2 + 3b\delta(\rho - \delta)];\ \eta\sqrt{\dfrac{2}{\epsilon}\left[b^2(\rho-\delta)^2 - \dfrac{b}{2}\right]}\right\}$,

---

[4] The complete calculations are available upon request to the authors.

then the profit function evaluated at P^* *is positive.*

Proof. A sufficient condition for the positivity of (34) is given by:

$$b^2(\rho-\delta)^2 - \frac{b}{2} - \frac{\epsilon\alpha^2}{2\eta^2} > 0,$$

which can be arranged by isolating α as in Proposition 1:

$$\alpha^2 < \frac{2\eta^2}{\epsilon}\left[b^2(\rho-\delta)^2 - \frac{b}{2}\right].$$

Thus, combining this condition with the one stated in Proposition 1 ensuring the existence and feasibility of P^*, we can conclude that if $b > \frac{1}{2(\rho-\delta)^2}$ and

$$\alpha < min\left\{\frac{\eta}{\delta A}[2+3b\delta(\rho-\delta)];\ \eta\sqrt{\frac{2}{\epsilon}\left[b^2(\rho-\delta)^2-\frac{b}{2}\right]}\right\},$$

then $\Pi^* > 0$. □

3.1.1 A numerical simulation

We chose the following values for parameters: $\delta = 0.01$, $\rho = 0.77$, $\epsilon = 0.05$, $A = 1$ $\eta = 0.001$, $b = 2.5$, verifying the conditions of Proposition 1. In particular, for $\alpha = 0.006$, P is feasible:

$$P = (0.9359,\ 0.9359,\ 0.0674,\ 0.0674,\ 0.0112,\ 0.0112,\ 0.0213,\ 0.0213).$$

Figure 1, sketched by employing Mathematica 5.0, is the outcome of a numerical simulation performed to illustrate the shape of $\Pi^*(\alpha)$. It shows that $\Pi^*(\alpha)$ keeps positive as $\alpha \in (0.002,\ 0.01)$.

It is worth noting that $\Pi^*(\alpha)$ is concave w.r.t. α, which can be explained on the basis of the balance between two opposite effects, i.e., the desirable gain generated by the transmission of technological knowledge through an increase in the spillover level, on the one hand, and the undesirable increase in the intensity of competition that the same fact brings about via a decrease in marginal costs and the resulting output expansion, on the other.

As we can infer from the simulation, $\Pi^*(\alpha)$ attains a maximum positive level and then decreases, becoming negative as α exceeds a certain threshold.

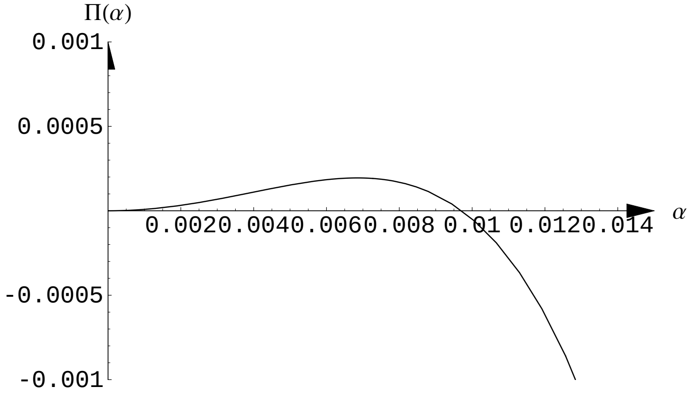

Figure 1: The graph of $\Pi^*(\alpha)$.

Substituting the values of parameters employed in Figure 1, the Jacobian matrix evaluated at the symmetric steady state reads as:

$$\mathcal{J}(P) = \begin{pmatrix} 0.01 & -0.0112 & -1.0674 & 0 \\ 0 & -0.001 & 0.006 & 0 \\ 0 & 0 & 0.76 & -0.4 \\ -0.0033 & 0.0037 & 0.3558 & 0 \end{pmatrix},$$

having the following eigenvalues:

$$\lambda_1 = 0, \qquad \lambda_2 = -0.01, \qquad \lambda_{3,4} = 0.385 \pm 0.0414i,$$

where $i = \sqrt{-1}$. Since λ_3 and λ_4 are complex conjugate with positive real parts, whereas λ_2 is negative, P turns out to be an unstable focus, with a 2-dimensional unstable manifold and a 1-dimensional stable manifold.

3.2 The asymmetric equilibrium structure

Here we focus on the possible arising of asymmetric outcomes. We are going to prove the following result:

Proposition 4. *If the following parametric hypotheses hold:*

1. $\rho > \delta$,

2. $b < \dfrac{1}{\delta(\rho-\delta)}$,

3. $\alpha \in \left(\dfrac{\eta[3+7b\delta(\rho-\delta)][1+b\delta(\rho-\delta)]}{4A\delta},\ \dfrac{\eta(1+3b\delta(\rho-\delta)+2(b\delta(\rho-\delta))^2)}{A\delta} \right)$,

the game also admits a non-symmetric steady state

$$Q = (c_1^{NS}, c_2^{NS}, \beta_1^{NS}, \beta_2^{NS}, k_1^{NS}, k_2^{NS}, q_1^{NS}, q_2^{NS}),$$

where:

$$c_1^{NS} = A - b(\rho-\delta)\left[\frac{3\eta[1+b\delta(\rho-\delta)] - \sqrt{\Omega(b,\eta,\delta,\rho,\alpha,A)}}{2\alpha}\right],$$

$$c_2^{NS} = A - b(\rho-\delta)\left[\frac{3\eta[1+b\delta(\rho-\delta)] + \sqrt{\Omega(b,\eta,\delta,\rho,\alpha,A)}}{2\alpha}\right],$$

$$\beta_1^{NS} = \frac{\eta[1+b\delta(\rho-\delta)] + \sqrt{\Omega(b,\eta,\delta,\rho,\alpha,A)}}{2\eta},$$

$$\beta_2^{NS} = \frac{\eta[1+b\delta(\rho-\delta)] - \sqrt{\Omega(b,\eta,\delta,\rho,\alpha,A)}}{2\eta},$$

$$k_1^{NS} = \frac{\eta[1+b\delta(\rho-\delta)] - \sqrt{\Omega(b,\eta,\delta,\rho,\alpha,A)}}{2\alpha},$$

$$k_2^{NS} = \frac{\eta[1+b\delta(\rho-\delta)] + \sqrt{\Omega(b,\eta,\delta,\rho,\alpha,A)}}{2\alpha},$$

$$q_1^{NS} = b(\rho-\delta)\left[\frac{\eta[1+b\delta(\rho-\delta)] - \sqrt{\Omega(b,\eta,\delta,\rho,\alpha,A)}}{2\alpha}\right].$$

$$q_2^{NS} = b(\rho-\delta)\left[\frac{\eta[1+b\delta(\rho-\delta)] + \sqrt{\Omega(b,\eta,\delta,\rho,\alpha,A)}}{2\alpha}\right],$$

and where

$$\Omega(b,\eta,\delta,\rho,\alpha,A) := \eta^2(1+b\delta(\rho-\delta))^2 - 4[(1+3b\delta(\rho-\delta)+2(b\delta(\rho-\delta))^2)\eta^2 - \alpha\eta\delta A].$$

Proof. Consider the non-symmetric case, consisting in the following relation between the *R&D* optimal strategies:

$$k_i = \frac{(1 + b\delta(\rho - \delta))\eta}{\alpha} - k_j.$$

We substitute k_i into (30) to obtain an equation having k_j as an unknown:

$$\begin{aligned} &- \frac{(1 + b\delta(\rho - \delta))\eta}{\alpha} + k_j - \frac{\alpha}{\eta} k_j^2 \\ &+ \delta \left[A - b(\rho - \delta)(k_j + 2\frac{(1 + b\delta(\rho - \delta))\eta}{\alpha} - 2k_j) \right] = 0, \end{aligned}$$

which can be arranged as follows:

$$\alpha^2 k_j^2 - \alpha\eta(1 + b\delta(\rho - \delta))k_j + [1 + 3b\delta(\rho - \delta) + 2(b\delta(\rho - \delta))^2]\eta^2 - \alpha\eta\delta A = 0.$$

The two roots of the latter equation are:

$$k_{1,2}^{NS} = \frac{\eta[1 + b\delta(\rho - \delta)] \pm \sqrt{\Omega(b, \eta, \delta, \rho, \alpha, A)}}{2\alpha}$$

where

$$\Omega(b, \eta, \delta, \rho, \alpha, A) := \eta^2(1+b\delta(\rho-\delta))^2-4[(1+3b\delta(\rho-\delta)+2(b\delta(\rho-\delta))^2)\eta^2-\alpha\eta\delta A].$$

To check whether the $k_{1,2}^{NS}$ are feasible we need to assess the positivity of the function $\Omega(b, \eta, \delta, \rho, \alpha, A)$:

$$\begin{gathered} \Omega(b, \eta, \delta, \rho, \alpha, A) > 0 \iff \\ \iff \eta^2(1 + b\delta(\rho - \delta))^2 > 4[(1 + 3b\delta(\rho - \delta) + 2(b\delta(\rho - \delta))^2)\eta^2 - \alpha\eta\delta A] \iff \\ \iff \ldots \iff -7(b\delta(\rho - \delta))^2 - 10b\delta(\rho - \delta) - 3 + \frac{4\alpha\delta A}{\eta} > 0, \end{gathered}$$

implying that α should exceed the level denoted by $\widehat{\alpha}$:

$$\alpha > \widehat{\alpha} \equiv \frac{\eta}{4A\delta}[(3 + 7b\delta(\rho - \delta))(1 + b\delta(\rho - \delta))]$$

As in the asymmetric case, the feasibility of the steady state requires that $0 < \beta_{1,2}^{NS} < 1$. Since obviously $\beta_1^{NS} > 0$ and $\beta_1^{NS} > \beta_2^{NS}$, we have to determine sufficient conditions such that the following system of inequalities holds:

$$\begin{cases} \beta_1^{NS} < 1 \\ \\ \beta_2^{NS} > 0 \end{cases} \iff \begin{cases} \sqrt{\Omega(\cdot)} < \eta[1 - b\delta(\rho - \delta)] \\ \\ \sqrt{\Omega(\cdot)} < \eta[1 + b\delta(\rho - \delta)] \end{cases},$$

hence the sufficient conditions can be expressed by the following system:

$$\begin{cases} b < \dfrac{1}{\delta(\rho-\delta)} \\ \\ \sqrt{\Omega(\cdot)} < \eta[1+b\delta(\rho-\delta)] \end{cases} .$$

Rearranging the latter inequality, we have that:

$$2(b\delta(\rho-\delta))^2 + 3b\delta(\rho-\delta) + 1 - \frac{\delta\alpha A}{\eta} > 0 \iff$$

$$\iff \dots \iff \alpha < \bar{\alpha} \equiv \frac{\eta(1+3b\delta(\rho-\delta)+2(b\delta(\rho-\delta))^2)}{A\delta}.$$

If we compare the two levels, a direct computation yields that $\widehat{\alpha} < \bar{\alpha}$ irrespective of all the remaining parameters' values, therefore a sufficient condition for α is $\alpha \in (\widehat{\alpha},\ \bar{\alpha})$. Combining this last constraint with the one for the feasibility of q_1^{NS} and q_2^{NS}, i.e. $\rho > \delta$, we obtain the three assumptions for all the coordinates of Q except c_1^{NS} and c_2^{NS}, whose expressions follow from the relations (26) and (27). Since $c_1^{NS} > c_2^{NS}$, it suffices to prove that $c_2^{NS} > 0$ under the same three assumptions. To begin with, we can rewrite it as follows:

$$c_2^{NS} > 0 \iff 2\alpha A - 3b\eta(\rho-\delta)[1+b\delta(\rho-\delta)] - b(\rho-\delta)\sqrt{\Omega(\cdot)} > 0.$$

Then, employing the above inequality

$$\sqrt{\Omega(\cdot)} < \eta[1+b\delta(\rho-\delta)] \iff -b(\rho-\delta)\sqrt{\Omega(\cdot)} > -\eta b(\rho-\delta)[1+b\delta(\rho-\delta)],$$

the previous expression can be estimated:

$$2\alpha A - 3b\eta(\rho-\delta)[1+b\delta(\rho-\delta)] - b(\rho-\delta)\sqrt{\Omega(\cdot)} >$$

$$> 2\alpha A - 3b\eta(\rho-\delta)[1+b\delta(\rho-\delta)] - \eta b(\rho-\delta)[1+b\delta(\rho-\delta)] =$$

$$= 2\alpha A - 4b\eta(\rho-\delta) - 2b^2\delta\eta(\rho-\delta)^2 > 0$$

if and only if the following condition on α holds:

$$\alpha > \widetilde{\alpha} := \frac{b\eta(\rho-\delta)[2+b\delta(\rho-\delta)]}{A}.$$

Consequently, now it is sufficient to prove that $\widetilde{\alpha} < \widehat{\alpha}$ in order that $\alpha \in (\widehat{\alpha}, \bar{\alpha})$ yields $c_2^{NS} > 0$. By using some algebra, we obtain that

$$\widetilde{\alpha} < \widehat{\alpha} \iff \frac{b\eta(\rho-\delta)[2+b\delta(\rho-\delta)]}{A} < \frac{\eta[3+7b\delta(\rho-\delta)][1+b\delta(\rho-\delta)]}{4A\delta} \iff$$

$$\iff \dots \iff 3+3b^2\delta^2(\rho-\delta)^2+2b\delta(\rho-\delta) > 0,$$

and this completes the proof of the feasibility of c_1^{NS} and c_2^{NS} and finally of the asymmetric steady state Q. $\square$

As we stated in Section 3, the linear dependence of $\dot{q}_1(t)$ and of $\dot{q}_2(t)$ on the remaining kinematic equations does not provide us with additional information on dynamics. Therefore, we are going to neglect them and construct the Jacobian matrix in the 6-equation case, evaluated at Q:

$$\mathcal{J}(Q) = \begin{pmatrix} \frac{\partial \dot{c}_1}{\partial c_1} & \frac{\partial \dot{c}_1}{\partial c_2} & \frac{\partial \dot{c}_1}{\partial \beta_1} & \frac{\partial \dot{c}_1}{\partial \beta_2} & \frac{\partial \dot{c}_1}{\partial k_1} & \frac{\partial \dot{c}_1}{\partial k_2} \\ \frac{\partial \dot{c}_2}{\partial c_1} & \frac{\partial \dot{c}_2}{\partial c_2} & \frac{\partial \dot{c}_2}{\partial \beta_1} & \frac{\partial \dot{c}_2}{\partial \beta_2} & \frac{\partial \dot{c}_2}{\partial k_1} & \frac{\partial \dot{c}_2}{\partial k_2} \\ \frac{\partial \dot{\beta}_1}{\partial c_1} & \frac{\partial \dot{\beta}_1}{\partial c_2} & \frac{\partial \dot{\beta}_1}{\partial \beta_1} & \frac{\partial \dot{\beta}_1}{\partial \beta_2} & \frac{\partial \dot{\beta}_1}{\partial k_1} & \frac{\partial \dot{\beta}_1}{\partial k_2} \\ \frac{\partial \dot{\beta}_2}{\partial c_1} & \frac{\partial \dot{\beta}_2}{\partial c_2} & \frac{\partial \dot{\beta}_2}{\partial \beta_1} & \frac{\partial \dot{\beta}_2}{\partial \beta_2} & \frac{\partial \dot{\beta}_2}{\partial k_1} & \frac{\partial \dot{\beta}_2}{\partial k_2} \\ \frac{\partial \dot{k}_1}{\partial c_1} & \frac{\partial \dot{k}_1}{\partial c_2} & \frac{\partial \dot{k}_1}{\partial \beta_1} & \frac{\partial \dot{k}_1}{\partial \beta_2} & \frac{\partial \dot{k}_1}{\partial k_1} & \frac{\partial \dot{k}_1}{\partial k_2} \\ \frac{\partial \dot{k}_2}{\partial c_1} & \frac{\partial \dot{k}_2}{\partial c_2} & \frac{\partial \dot{k}_2}{\partial \beta_1} & \frac{\partial \dot{k}_2}{\partial \beta_2} & \frac{\partial \dot{k}_2}{\partial k_1} & \frac{\partial \dot{k}_2}{\partial k_2} \end{pmatrix} =$$

$$= \begin{pmatrix} \delta & 0 & -k_2^{NS} & 0 & -1 & -\beta_1^{NS} \\ 0 & \delta & 0 & -k_1^{NS} & -\beta_2^{NS} & -1 \\ 0 & 0 & -\eta & 0 & 0 & \alpha \\ 0 & 0 & 0 & -\eta & \alpha & 0 \\ 0 & 0 & 0 & 0 & \rho-\delta & 0 \\ 0 & 0 & 0 & 0 & 0 & \rho-\delta \end{pmatrix} \tag{35}$$

Because (35) is an upper triangular matrix, so its entries on the main diagonal are its eigenvalues. It is easy to prove the following:

Proposition 5. *The asymmetric steady state Q is a saddle point equilibrium.*

Proof. By the same procedure carried out in Proposition 2, the eigenvalues of (35) are δ, $-\eta$ ad $\rho - \delta$, having algebraic multiplicity 2. Hence, Q is a saddle point whose stable manifold has dimension 2 and whose unstable manifold has dimension 4. $\square$

The profits of firm i at asymmetric equilibrium are:

$$\Pi_i^{NS} = \left[A - q_i^{NS} - q_j^{NS} - c_i^{NS}\right] q_i^{NS} - \frac{b(k_i^{NS})^2}{2} - \frac{\epsilon(\beta_i^{NS})^2}{2}. \tag{36}$$

In order to check that $\Pi_i^{NS} > 0$ for both firms, when the asymmetric equilibrium is feasible, we shall prove the following Proposition:

Proposition 6. *If*

1. *Proposition 4 holds with the further hypothesis* $\rho > \dfrac{3\delta}{2}$*;*
2. $b > \dfrac{1}{2(\rho-\delta)^2}$*;*
3. $\epsilon < \dfrac{b(2b(\rho-\delta)^2-1)\eta^2}{\alpha^2}\left(\dfrac{\eta(1+b\delta(\rho-\delta)) - \sqrt{\Omega(\cdot)}}{\eta(1+b\delta(\rho-\delta)) + \sqrt{\Omega(\cdot)}}\right)^2,$

then both players'profits are positive at the steady state Q.

Proof. Plugging the coordinates of Q into the expressions (36) and imposing positivity, we obtain two different inequalities which can be expressed by isolating parameter ϵ, i.e.:

$$\Pi_1^{NS} > 0 \iff \epsilon < \frac{b(2b(\rho-\delta)^2-1)\eta^2}{\alpha^2}\left(\frac{\eta(1+b\delta(\rho-\delta)) - \sqrt{\Omega(\cdot)}}{\eta(1+b\delta(\rho-\delta)) + \sqrt{\Omega(\cdot)}}\right)^2, \tag{37}$$

$$\Pi_2^{NS} > 0 \iff \epsilon < \frac{b(2b(\rho-\delta)^2-1)\eta^2}{\alpha^2}\left(\frac{\eta(1+b\delta(\rho-\delta)) + \sqrt{\Omega(\cdot)}}{\eta(1+b\delta(\rho-\delta)) - \sqrt{\Omega(\cdot)}}\right)^2, \tag{38}$$

where $\Omega(b, \eta, \delta, \rho, \alpha, A)$ is defined as in Proposition 4.

Since $\epsilon > 0$, a necessary condition for (37) and (38) to hold is given by: $b > \dfrac{1}{2(\rho - \delta)^2}$, which must be compliant with the assumption on b of Proposition 4. That can occur if we replace the assumption $\rho > \delta$ with $\rho > \dfrac{3\delta}{2}$.

Since the right hand side of (38) is larger than the right hand side of (37) irrespective of all parameters' values, the most restrictive inequality is (37). Hence, combining all the previous parametric assumptions, the positivity of both profits is verified. □

3.2.1 A numerical simulation

Also in this case, we may carry out some numerical simulations for illustrative purposes. Choosing the parameter values $\delta = 0.05$, $\rho = 0.5$, $\epsilon = 1.2$, $A = 1$, $b = 2.5$, $\eta = 0.0005$, $\alpha = 0.011$, we can list the equilibrium levels of states, controls and profits in the next table and proceed to a comparison between the players' performances:

	c^{NS}	β^{NS}	k^{NS}	q^{NS}	Π^{NS}
1st firm	0.942068	0.979609	0.00348368	0.00391914	0.575781
2nd firm	0.895894	0.0766409	0.0445277	0.0500936	0.00355528

Figure 2 illustrates the behavior of the two firms' profits as functions of α. A general appraisal of the comparative performance of firms is that firm 1 attains higher profits by virtue of the following mechanism: a lower R&D effort yields a higher production cost, which in turn brings about an output restriction; hence, firm 1 essentially aims at reducing its own investment costs while free-riding over the rival's R&D activity. Overall, the cost-saving effect of shrinking the R&D investment more than offset the negative consequences of operating at a higher marginal cost and selling a lower quantity (which always amounts to bad news under Cournot competition).

$\mathcal{J}(Q)$ has the following eigenvalues: $\delta = 0.05$, $-\eta = -0.0005$, $\rho - \delta = 0.45$, all of them being double roots of the associated characteristic polynomial. By Proposition 5, Q is a saddle point too, hence as in the symmetric case there exist optimal trajectories heading towards Q.

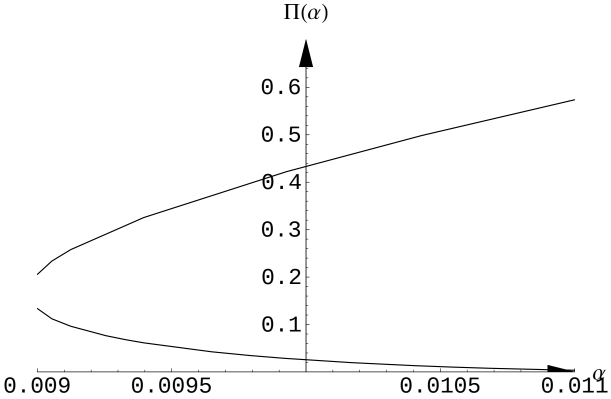

Figure 2: The upper graph represents $\Pi_1^*(\alpha)$ and the lower one represents $\Pi_2^*(\alpha)$ as $\alpha \in (0.009,\ 0.011)$. In this range of parameters, the difference between profits (as well as the prevalence of Π_1^*) is particularly evident.

4 Concluding remarks

This paper presents a model of dynamic Cuornot duopoly where the effects of a spillover due to Open Innovation is taken into account. We described it in a differential game with cost-reducing R&D. The main goal was to investigate the nature of Open Innovation in a small oligopoly and make an analogy to extant economical and managerial issues. To do so, we have nested the endogeneity of knowledge spillovers into the setup dating back to Cellini and Lambertini (2005, 2009), transforming the spillover parameter into a state variable evolving endogenously under the effect of external R&D. This approach seems to be more realistic, in that the OI flow is dynamically driven by R&D evolution, as all the real world examples suggest.

By doing so, we have identified the open-loop equilibrium structure, where we have achieved multiple equilibria, with both symmetric and asymmetric steady states. In both cases, we have found the parametric conditions under which the steady states are feasible and the firms' profits are positive.

In the symmetric steady state solution, a numerical simulation shows that the equilibrium profit attains a unique maximum value corresponding to a growth rate of knowledge spillover under R&D effect.

On the other hand, the asymmetric solution is quite interesting as it is generated by a setup which, a priori, is fully symmetric. Numerical simulations show that the firm with a higher private R&D investment level has a considerably smaller level of OI absorption effort, and *vice versa*. Moreover, profits increase as OI absorption increases, even in presence of a lower level of production.

In view of the growing relevance of OI, research on this issue will plausibly intensify in the near future. Possible developments include investigating (i) the possibility of selling R&D spillovers in the market, taking into account the issue of property rights; and (ii) determining the feedback equilibrium structure.

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